**Journal of Electrical Engineering, Electronics, Control and Computer Science – JEEECCS, Volume 8, Issue 28, pages 29-38, 2022**

# Fuzzy-FOPIDF control for tracking the trajectory of nonholonomic Wheeled Mobile Robot

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***Abstract*** **– This paper proposes a Fuzzy Fractional-Order Proportional Integral and Derivate Controller (Fuzzy-FOPIDF) for trajectory tracking of a non-holonomic wheeled mobile robot. The proposed Fuzzy-FOPIDF controller consists of a Fractional-Order Proportional Integral and Derivate (FOPIDF) controller and a Fuzzy Logic Controller (FLC) where the FLC is used to dynamically determine the $K_P$ , $K_I$ and $K_D$ parameters of the FOPIDF. Five different trajectories including linear trajectory, L-shaped trajectory, sinusoidal trajectory, circular trajectory and Lemniscate trajectory are used to evaluate the performances of the proposed controller. These performances are compared with those of Fuzzy-PID, PID and FOPID controllers based on the results of computer simulations performed with MATLAB/Simulink software. These simulation results show that, better performance in terms of convergence, stability, speed and tracking accuracy are obtained with the proposed controller compared to other controllers.**

**Keywords–Fuzzy-FOPIDF controller; Trajectory tracking; nonholonomic Wheeled Mobile Robot.**

## I. INTRODUCTION

In the last decades, with the continuous development of technology in the world, mobile robotics has been the focus of considerable attention in various fields of industry. The design of efficient control technology is a complex activity due to the non-ergonomic nature of mobile robotic systems. It is also important to remember that the wheeled mobile robot experiences unknown internal and external disturbances while moving in a real environment. The main problem in mobile robotics is the control of stabilization and trajectory tracking. Trajectory tracking control ensures that the robot follows a desired trajectory that is governed by geometric parameters [1-3]. In motion control in mobile robotics, trajectory tracking control is essential because, the system must move to a specific point in a specific time. In recent years, a multitude of controllers have been proposed to solve these problems [4-8]. To compensate for the unknown nonlinear terms, a homogeneous high gain observer is proposed in [9-10], here the system states are estimated from new dynamic gains. The linearized system being controllable in a general way on the whole reference trajectory. In [11], the authors have developed a linearization-based supervisory controller for systems with nonholonomic constraints. A sliding mode control strategy based on two-dimensional polar kinematics for mobile robots is proposed in [12]. In order to overcome the problem of singularities arising from polar coordinates, a kinematic model in Cartesian coordinates has been presented in [13]. In [14], the authors have proposed a classical kinematic stability control technique for nonholonomic robots based on the Lyapunov method. In [15], the description of a nonlinear system perturbed by the integration of an observer-based control technique is proposed. In [16], the authors propose a new observer-based control technique for training follower leaders and the efficiency, robustness and applicability in several cases have been demonstrated. In [17], the authors have developed a design model of an adaptive virtual speed controller and a torque control strategy for the robot with external disturbances and uncertain variables. In [18], the non-stationary cascade system, grouping two uniformly stable subsystems of finite duration is studied by the authors.

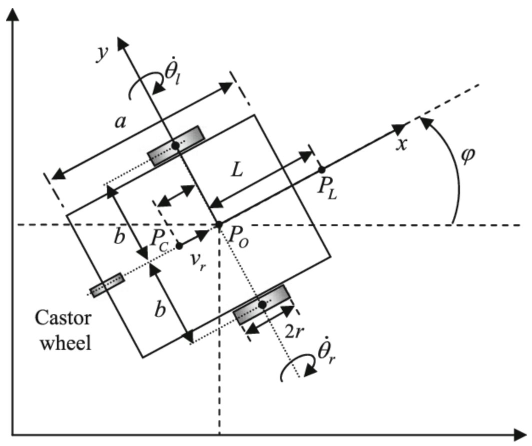
Given the complexity of accurately obtaining the upper bound of the localized disturbance in the real domain, it is necessary to design a controller to eliminate localized disturbances. The authors of [19] propose a dynamic adaptive control strategy for a nonholonomic mobile robot equipped with uncertain dynamic variables. The adaptive control strategy for the mobile robot is obtained by using the recoil technology. Faced with the problem of tracking and stabilizing an internally damped mobile robot with undefined parameters and saturation disturbance of the external input torque, the authors of [20] have developed a new adaptive scheme. The authors of [21] developed an adaptive super-twisting algorithm to follow a predetermined path that ignores slamming. In order to transgress this limitation, the terminal sliding mode has been used in many control applications. A design strategy for terminal sliding mode control for nonlinear systems with feedback was developed in [22]. The authors of [23] present a fast finite-time state and observer designed from a fast finite-time control algorithm. The authors of [24] propose the sliding mode control (SMC) problem with output feedback for uncertain discrete-time nonlinear systems using the fuzzy dynamics TS model. An adaptive unit-vector sliding mode control technique based on monitoring functions to solve the unknown boundary disturbance problem is presented in [25]. The authors of [26] propose a novel time-shifting technique for actuator abnormalities regeneration of systems with output delay based on a sliding mode observer. Adaptive high-gain stabilizers for a group of linear time-invariant state-space systems have been exposed in [27]. Nevertheless, there are still problems like input saturation, input deadband, constraints and many others. The authors of [28] gave the alternative in uninterrupted tuning for pneumatic artificial muscle systems. The authors of [29] propose an adaptive law available to overcome the uncertainties of variables and armature for embedded winch systems. Artificial neurons have the ability to approximate highly nonlinear and difficult systems when the uncertainties of the unmodeled parameters and the nonparametric uncertainty of the non-holonomic wheeled mobile system are taken into account [30]. In [31], an adaptive neural slip mode controller is presented for non-holonomic wheeled mobile systems with model uncertainties and external parasitics. The input torque of the robot can be extended to the so-called dead part by [32]. The authors of [33] propose a new neural network based adaptive tracking controller, in which the neural network is used to overcome the uncertainty caused by wheel slip and external force to achieve the desired tracking efficiency. Among several control methods, the PID controller is widely used in the field of robots. By using the PID controller, both stability and tracking control can be achieved, but the accuracy is not high [34]. Fuzzy logic control of autonomous vehicles has gained popularity in the last few decades due to its very successful implementation in almost every field. This has led to the design of a fuzzy PID controller.

In this paper, a fractional order PIDF fuzzy controller (Fuzzy-FOPIDF) for tracking the trajectory of a unicycle mobile robot with differential wheels is proposed. The model of a differential drive mobile robot based on the Lagrange dynamic approach is described [35]. The fractional order PIDF fuzzy controller (Fuzzy-FOPIDF), the PID fuzzy controller (Fuzzy-PID), the PID controller and the fractional order PID controller are compared by the simulation.

## II. MODELING OF THE MOBILE ROBOT WITH DIFFERENTIAL WHEELS

A unicycle robot is a mobile robotic system with differential drive and in its structure, the robot is equipped with two independent motors that connect two wheels on a common axis. The model of mobile robot with differential drive includes a chassis with two wheels of radius "r", placed at a distance "a" from the center of the robot, assembled on a common axis and possibly including an idler wheel to ensure its stability. Its center of rotation is located on the axis connecting the two driving wheels. [1].

Figure 1. Mobile robot model with differential drive [36]




### A. Kinematic Model

Robot kinematics usually have two analyzes main, direct kinematics and inverse kinematics. The problem of direct kinematics is described by the following function:

$$\dot{q} = \left[\dot{x}_g \quad \dot{y}_g \quad \dot{\varphi} \quad \dot{\theta}_r \quad \dot{\theta}_l\right] = f\left(\dot{\theta}_r, \dot{\theta}_l, b, r, \varphi\right) \quad (1)$$

where

$\dot{q}$ is kinematic model output vector

$x_g$ , $y_g$ are the coordinates of the center of masse of the robot

$\varphi$ is the rotation angle of the platform

$\dot{\theta}_r$ is the angular velocity of the right wheel

$\dot{\theta}_l$ is the angular velocity of the left wheel

$b$ is the distance between each driving wheel and the axis of symmetry robot in the Y direction.

$r$ is the radius of each drive wheel

The kinematic model of the robot is given by [35]:

$$\begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{\varphi} \\ \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\varphi + \frac{br}{2b}\sin\varphi & \frac{r}{2}\cos\varphi - \frac{br}{2b}\sin\varphi \\ \frac{r}{2}\sin\varphi + \frac{br}{2b}\cos\varphi & \frac{r}{2}\sin\varphi - \frac{br}{2b}\cos\varphi \\ \frac{r}{2b} & -\frac{r}{2b} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} \quad (2)$$

Another form of direct kinematics is given by [35]:

$$\begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{\varphi} \\ \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & \cos\varphi \\ 0 & 1 \\ \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (3)$$

where:

$v_r = r\dot{\theta}_r$ : Linear speed of the right wheel

$v_l = r\dot{\theta}_l$ : Linear speed of the left wheel

$v = r\dot{\varphi}$ : Linear speed of the robot

The inverse kinematics is described as follows [35]:

$$\begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} = \frac{1}{r}\begin{bmatrix} \cos\varphi & \sin\varphi & b & 1 & 0 \\ \cos\varphi & \sin\varphi & -b & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{\varphi} \\ \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} \quad (4)$$

The motion of a differential mobile robot is characterized by two constraint equations non-holonomic, which are obtained by two main assumptions which represent the non-holonomic constraints, and which are written in more compact form like:

$$A(q)\dot{q} = 0 \quad (5)$$

with

$$A(q) = \begin{bmatrix} \cos\varphi & \sin\varphi & b & -r & 0 \\ \cos\varphi & \sin\varphi & -b & 0 & -r \\ -\sin\varphi & \cos\varphi & a & 0 & r \end{bmatrix}$$

*B. Dynamic Model*

In this study, the dynamic model is developed using the Lagrange-Euler formalism. Considering the rolling of the wheels subjected to lateral and longitudinal slippage, the dynamic model of the robot is given by [35]:

$$\dot{\upsilon} = \left(S^T(q)M(q)S(q)\right)^{-1} \begin{pmatrix} S^T(q)B(q)\tau + \\ S^T(q)F(\dot{q}) - \\ S^T(q)M(q)\dot{S}(q)\upsilon - \\ S^T(q)C(q,\dot{q}) \end{pmatrix} \quad (6)$$

where

$M(q)$: is the inertia matrix of the system

$C(q,\dot{q})$: is the vector of the centrifugal and Coriolis force

$B(q)$: is the input transformation matrix

$F(\dot{q})$: is the vector of traction forces

$\tau$ : is the vector motor torque acting on each axis of the wheel generated by its DC motor.

with

$$M(q) = diag\begin{bmatrix} m_r \\ m_l \\ I_{rz} + 2I_{wz} \\ m_w \\ m_w \\ m_w \\ m_w \\ I_{wy} \\ I_{wy} \end{bmatrix}, C(q,\dot{q}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ m_w\dot{\varphi}\dot{\rho}_r \\ m_w\dot{\varphi}\dot{\rho}_l \\ -m_w\dot{\varphi}\dot{\eta}_r \\ -m_w\dot{\varphi}\dot{\eta}_l \\ 0 \\ 0 \end{bmatrix} \quad \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}$$

$$F(\dot{q}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ f_{latr} \\ f_{latl} \\ f_{longr} \\ f_{longl} \\ -rf_{longr} \\ -rf_{longl} \end{bmatrix}, \; B(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \; \upsilon = \begin{bmatrix} \eta_l \\ \rho_r \\ \rho_l \\ \theta_r \\ \theta_l \end{bmatrix},$$

$$S(q) = \begin{bmatrix} -\sin\varphi & \frac{a\cos\varphi + b\sin\varphi}{2a} & \frac{a\cos\varphi - b\sin\varphi}{2a} & 0 & 0 \\ -\sin\varphi & \frac{a\sin\varphi - b\cos\varphi}{2a} & \frac{a\sin\varphi + b\cos\varphi}{2a} & 0 & 0 \\ 0 & \frac{1}{2a} & -\frac{1}{2a} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The dynamic parameters and variables of the mobile robot are defined as follows:

$\rho - \eta$ : The coordinate system fixes at each drive wheel center.

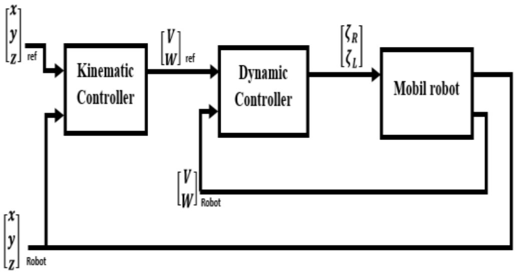
$f_{latr,l}$ : The lateral pulling force for each driving wheel.

$f_{longr,l}$ : The longitudinal traction force for each driving wheel.

## III. CONTROLLER DESIGN

The controller design has two parts: The Kinematic Controller which is based on inverse kinematics and the Dynamic Controller which is a fuzzy fractional order PID controller. In order to ensure better trajectory following, we have developed a law made up of two loops nested one inside the other. The internal loop representing the dynamic part (Fuzzy-FOPID) of the robot and the external loop representing the kinematic part of the robot.

Figure 2. Block diagram of robot’s controller




### A. Kinematic Controller

The kinematic controller received the desired and measured values of position and returns the desired linear velocity $\left(v_{ref}\right)$ and angular velocity $\left(\omega_{ref}\right)$. The aim of this controller is to converge the errors of the local robot frame $e_x$, $e_y$ and $e_\varphi$ to 0 and to stabilize them at this value.

The errors expressed in the local robot frame are given by [35]:

$$\begin{bmatrix} e_x(t) \\ e_y(t) \\ e_\varphi(t) \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d - x_g \\ y_d - y_g \\ \varphi_d - \varphi_g \end{bmatrix} \quad (7)$$

The controller of the kinematic stabilization being based on the error dynamics, the Lyapunov method for the calculation of linear and angular velocities is applied. the candidate Lyapunov function given by eq. (8) verifying the three properties of Lyapunov functions is used.

$$V = \frac{1}{2}\left(e_x^2 + e_y^2\right) + \frac{1}{k_y}\left(1 - \cos\left(e_\varphi\right)\right) \quad (8)$$

with $k_y > 0$

Consider a mobile robot represented by a nonlinear model of the form:

$$\dot{q} = f\left(q, u\right) \quad (9)$$

we obtain:

$$\dot{V} = \left(v_d \cos\varphi_d - v\right)e_x + \frac{1}{k_y}\left(k_y v_d e_y + \omega_d - \omega\right)\sin\left(e_\varphi\right) \quad (10)$$

To give back

$$\dot{V} = k_y e_x^2 - \frac{k_\varphi}{k_y}\sin^2\left(e_\varphi\right) \quad (11)$$

Which leads to:

$$\begin{cases} v_{ref} = v_d \cos e_\varphi + k_x e_x \\ \omega_{ref} = k_\varphi \sin e_\varphi + k_y v_d e_y + \omega_d \end{cases} \quad (12)$$

where:

$k_x$, $k_y$ and $k_\varphi$ are positive gains.

### B. Dynamic Controller

The dynamic part of controller is a fuzzy fractional order PID (FFOPIDF). This controller self-adjusting designed contains two parts: a Fractional order PID and a fuzzy logic controller (FLC), as shown in Fig. 3. The fuzzy logic controller is used to regulate the parameters $K_P$, $K_I$ and $K_D$ of the controller Fractional order PID while FOPIDF parameters are selected by trial-and-error method.

The dynamic controller received the desired and measured values of velocity and returns the motor torque acting on each axis of the wheel generated by its DC motor.

Figure 3. Fuzzy fractional-order PID structure

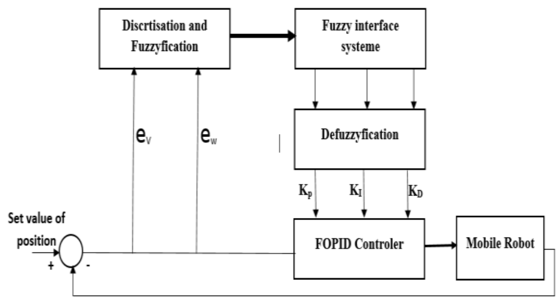
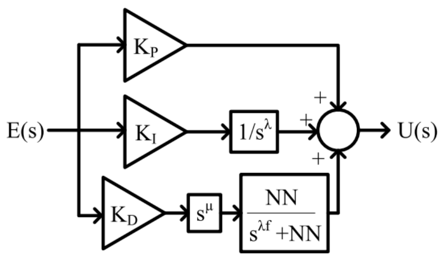



Figure 4. FOPIDF structure used [37]




Mamdani type fuzzy inference mechanism is applied to the proposed fuzzy controller used. It receives as input the errors of the linear and angular velocities and returns as output the coefficients $K_P, K_I$ and $K_D$ of the fractional order PIDF controller whose structure is given in Fig. 4.

The choice of triangular membership functions was done expertly. The AND function has been integrated into the choice of fuzzy rules.

The membership functions of inputs $e_v$ and $e_\omega$, outputs $K_P, K_I$ and $K_D$, fuzzy rules are given in table 1 and Fig. 5.

The transfer function of the FOPIDF regulator is given by :

$$G_{(FOPIDF)} = k_P + \frac{1}{s^{\lambda}} k_I + s^{\mu} \frac{NN}{s^{\lambda_f} + NN} k_D \qquad (13)$$

According to [37], the stability of the dynamic controller Fuzzy-FOPIDF,in the sense of Lyapunov,is based on the positivity of the gains resulting from the transfer function of the FOPIDF controller by also considering the positivity of the fractional orders.

$$k_P > 0 \quad ; \quad k_I > 0 \ ; \ k_D > 0$$

Figure 5. Membership functions of input and output.

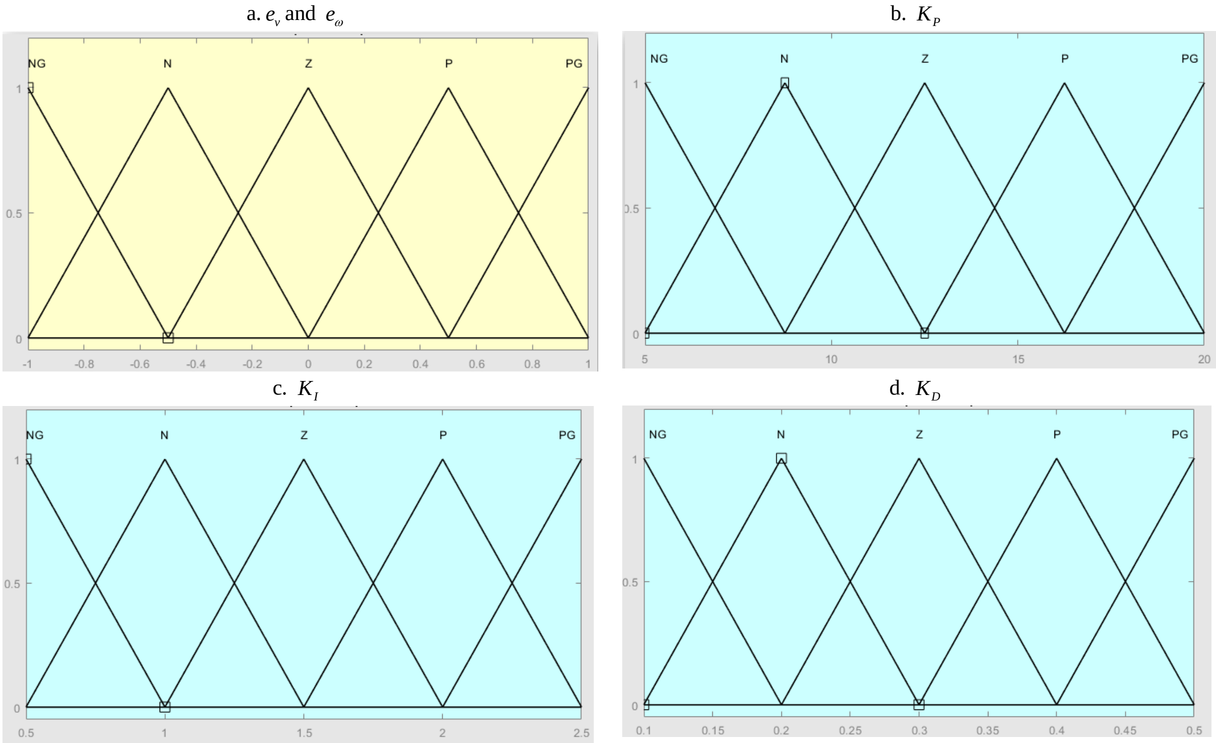



TABLE I. FUZZY RULE TABLE FOR $K_P/K_I/K_D$

| $e_v$ / $e_\omega$ | ***NG*** | ***N*** | ***Z*** | ***P*** | ***PG*** |
|---|---|---|---|---|---|
| ***NG*** | NG/NG /PG | NG/NG /PG | NG/NG /PG | N/N/P | Z/Z/Z |
| ***N*** | NG/NG /PG | NG/NG /PG | N/N/P | Z/Z/Z | P/P/N |
| ***Z*** | NG/NG /PG | N/N/P | Z/Z/Z | P/P/N | PG/PG/ NG |
| ***P*** | N/N/P | Z/Z/Z | P/P/N | PG/PG/ NG | PG/PG/ NG |
| ***PG*** | Z/Z/Z | P/P/N | PG/PG/ NG | PG/PG/ NG | PG/PG/ NG |

## IV. RESULTS AND DISCUSSION

MATLAB environment is used as an experimental platform to carry out simulations on the robot model. The real technical characteristics are from Pioneer 3DX robot [35]. The tracking results of the proposed controller for different trajectories are presented below and compared to those obtained with the PID, FOPID and Fuzzy PID controllers. The values of the kinematic controller gains used for the different controllers are: $k_x = 1.5$, $k_y = 20$ and $k_\varphi = 50$

TABLE II. ROBOT'S PARAMETERS VALUES

| **Symbol** | $a$ | $b$ | $r$ | $I_{rz}$ | $I_{wy}$ | $I_{wz}$ |
|---|---|---|---|---|---|---|
| **value** | 0.05 (m) | 0.24 (m) | 0.095 (m) | 0.537 (kg.m$^2$) | 0.0023 (kg.m$^2$) | 0.0011 (kg.m$^2$) |
| **Symbol** | $m_r$ | $m_w$ | $f_{longr}$ | $f_{longl}$ | $f_{latr}$ | $f_{latl}$ |
| **value** | 17 (kg) | 0.05 (kg) | 0.021 (N) | 0.15 (N) | 0.1545 (N) | 0.45 (N) |

The parameters of the different controllers are given in Table III:

TABLE III. CONTROLLERS PARAMETERS

| | **Parameters** | | | | | |
|---|---|---|---|---|---|---|
| **controller** | $K_P$ | $K_I$ | $K_D$ | $\mu$ | $\lambda$ | $\lambda_f$ |
| PID | 10 | 1.5 | 0.35 | / | / | / |
| FOPID | 10 | 1.5 | 0.35 | 0.5 | 0.85 | 0.75 |
| Fuzzy-PID | / | / | / | / | / | / |

| | Parameters | | | | | |
|---|---|---|---|---|---|---|
| **controller** | $K_P$ | $K_I$ | $K_D$ | $\mu$ | $\lambda$ | $\lambda_f$ |
| PID | 10 | 1.5 | 0.35 | / | / | / |
| Fuzzy FOPID | / | / | / | 0.5 | 0.85 | 0.75 |

## A. *Linear-shaped Trajectory*

the equation of the linear trajectory used is:

$$\begin{cases} x_d = 0.3t \\ y_d = 0.3t \\ \varphi_d = \pi / 4 \end{cases} \tag{14}$$

Figure 6. trajectory along the x and y axes

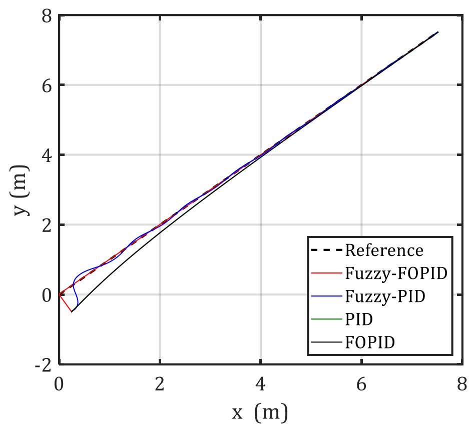



Figure 7. Position x axis and its error

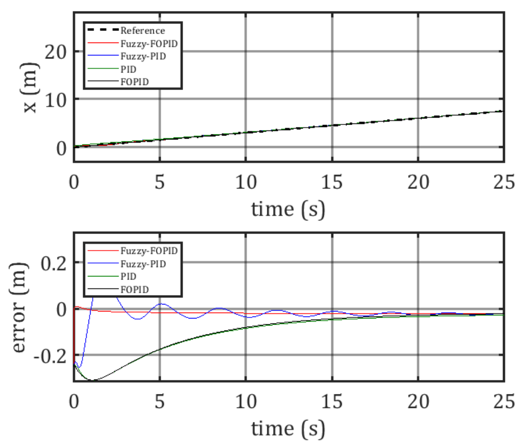
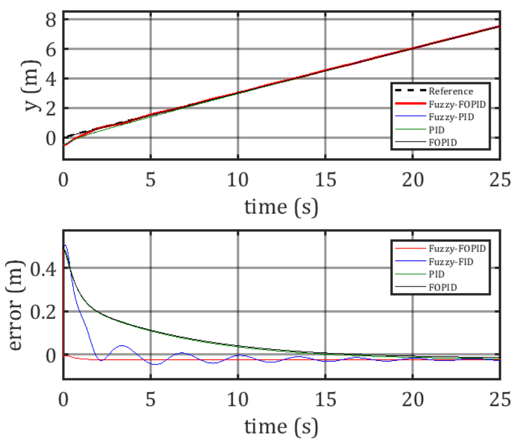



Figure 8. position y axis and its error




## B. *L-shaped trajectory*

The equation of the linear trajectory used is:

$$\begin{cases} x_d = t, y_d = \varphi_d = 0 \quad if \quad 0 \le t < 3 \\ x_d = 3, y_d = t - 3, \varphi_d = \dfrac{\pi}{2} \quad if \quad t \ge 3 \end{cases} \tag{15}$$

Figure 9. trajectory along the x and y axes

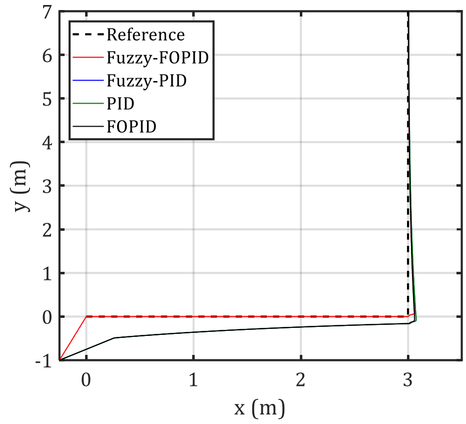



Figure 10. Position x axis and its error

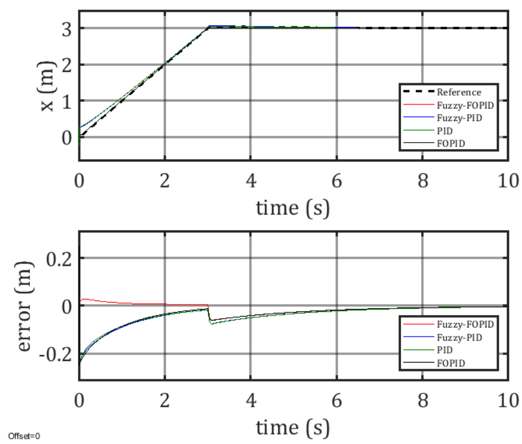
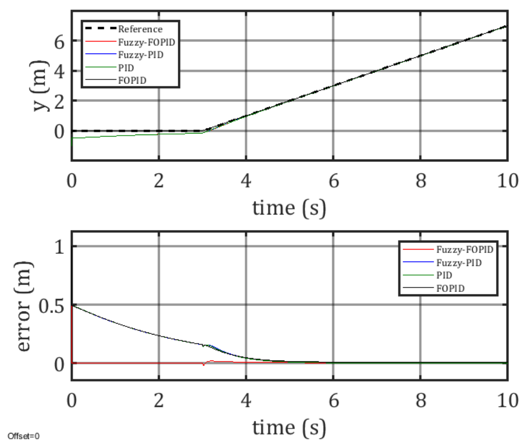



Figure 11. position y axis and its error



### *C. Sinusoidal-shaped Trajectory*

The equation of the linear trajectory used is:

$$\begin{cases} x_d = t \\ y_d = 0,2 + 0.25\sin\left(0.3\pi t\right) \\ \varphi_d = \tan^{-1}\left(\dot{y}_d / \dot{x}_d\right) \end{cases} \qquad (16)$$

Figure 12. trajectory along the x and y axes

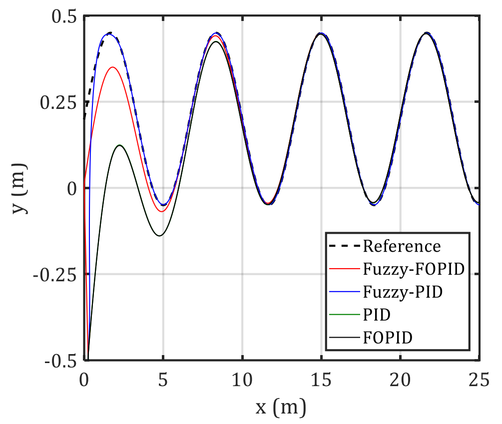



Figure 13. Position x axis and its error

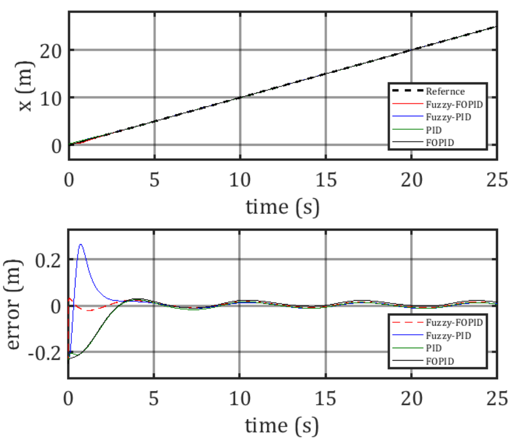
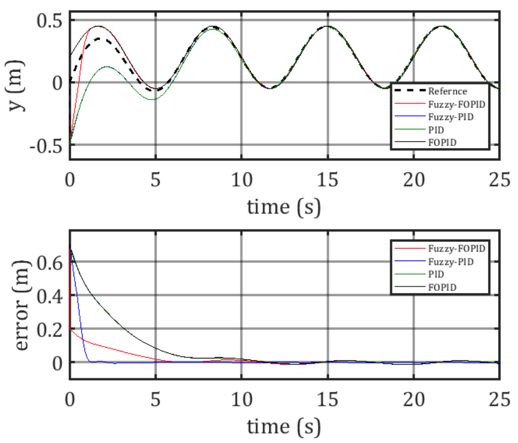



Figure 14. Position y axis and its error




### *D. Circular-shaped Trajectory*

The equation of the linear trajectory used is:

$$\begin{cases} x_d = \left(v_d / \omega_d\right)\cos\left(\omega_d t - 0,5\pi\right) \\ y_d = \left(v_d / \omega_d\right)\cos\left(\omega_d t - 0,5\pi\right) \\ \varphi_d = \tan^{-1}\left(\dot{y}_d / \dot{x}_d\right) \end{cases} \qquad (17)$$

Figure 15. trajectory along the x and y axes

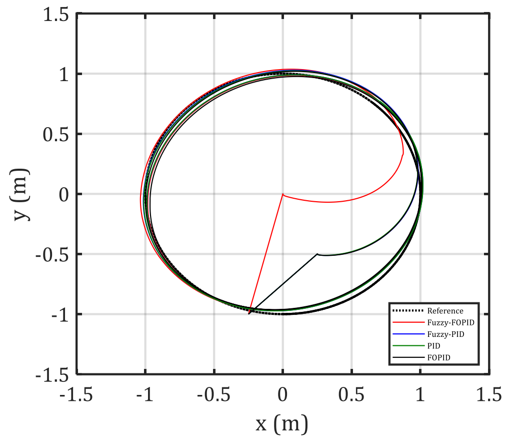



Figure 16. Position x axis and its error

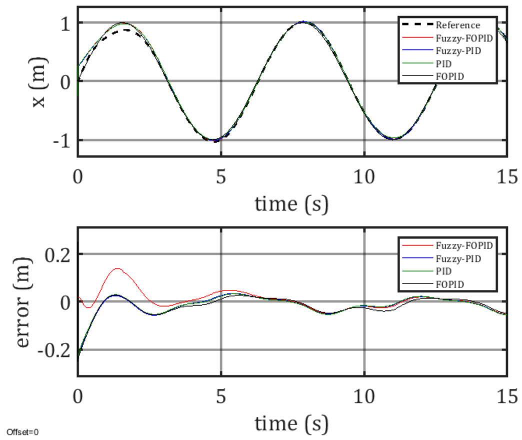
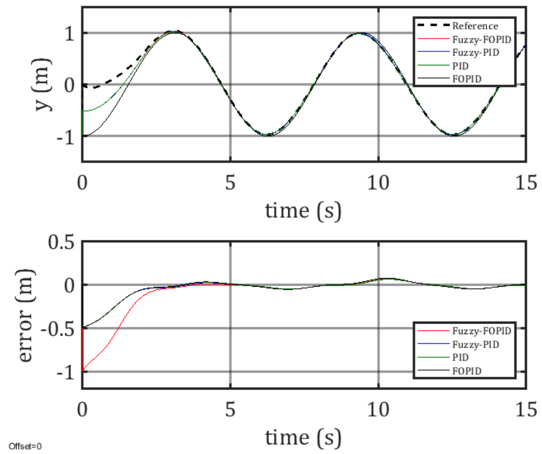



Figure 17. Position y axis and its error



### *E. Lemniscate-shaped Trajectory*

The equation of the linear trajectory used is:

$$\begin{cases} x_d = \sin\left(2\pi t / 50\right) \\ y_d = \sin\left(2\pi t / 50\right) \\ \varphi_d = \tan^{-1}\left(\dot{y}_d / \dot{x}_d\right) \end{cases} \quad (18)$$

Figure 18. trajectory along the x and y axes

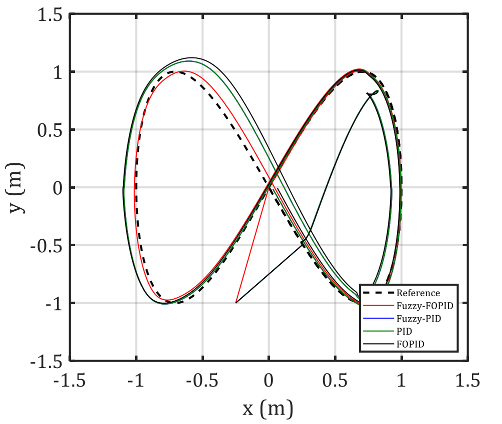



Figure 19. Position x axis and its error

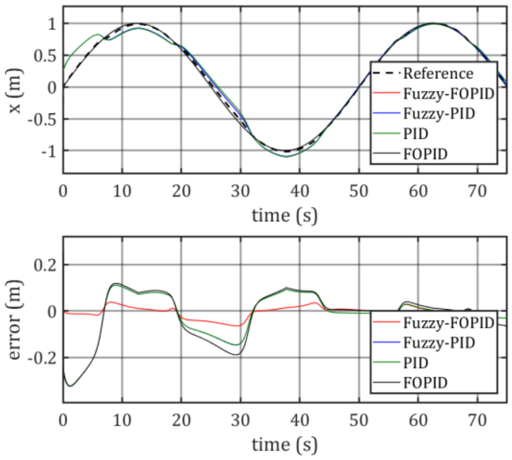
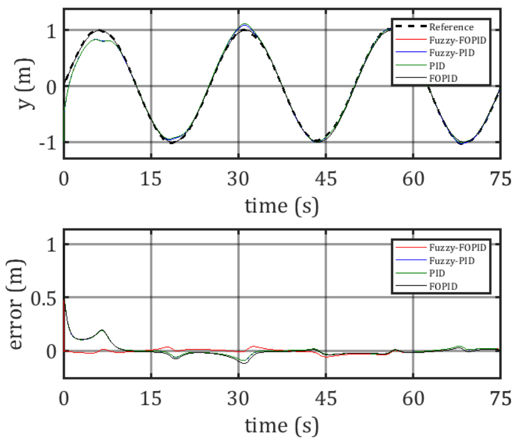



Figure 20. Position y axis and its error




The Fig. 6, 7 and 8 represent the tracking of a linear trajectory by the mobile robot by applying the proposed fuzzy-FOPIDF controller and Fuzzy-PID, PID and FOPID controllers. we can see that the fuzzy-FOPID controller provides a better trajectory tracking. The trajectory tracking error site by the fuzzy-FOPID controller reaches a much lower value with a very small response time compared with the other controllers. The Fig. 9,10 and 11 represent the tracking of a L-shaped trajectory by the mobile robot by applying the proposed fuzzy-FOPID controller and Fuzzy-PID, PID and FOPID controllers. we can see that the fuzzy-FOPID controller provides a better trajectory tracking. The trajectory tracking error site by the fuzzy-FOPID controller reaches a much lower value with a very small response time compared with the other controllers. From Fig. 12, 13 and 14, it can be seen that, the distance error of the mobile robot on a sinusoidal trajectory by the Fuzzy-FOPID controller is relatively small compared to the other controllers, but with a slightly small response time compared to the Fuzzy-PID controller. The Fig. 15, 16 and 17 represent the tracking of a circular trajectory by the mobile robot by applying the proposed fuzzy-FOPID controller and Fuzzy-PID, PID and FOPID controllers. We can see that the fuzzy-FOPID controller provides a better trajectory tracking. The trajectory tracking error site by the fuzzy-FOPID controller reaches a much lower value with a very small response time compared with the other controllers. The Fig. 18, 19 and 20 represent the tracking of a lemniscate-shaped trajectory by the mobile robot by applying the proposed fuzzy-FOPID controller and Fuzzy-PID, PID and FOPID controllers. We can see that the fuzzy-FOPID controller provides a better trajectory tracking. The trajectory tracking error site by the fuzzy-FOPID controller reaches a much lower value with a very small response time compared with the other controllers.

The simulation results show that the Fuzzy-FOPID provides better trajectory tracking compared to the Fuzzy-PID, PID and FOPID controllers with relatively smaller errors.

Compared to previously proposed control techniques for tracking the trajectory of nonholonomic Wheeled Mobile Robot [33,38]; While recalling that our system which takes into account the rolling of the wheels subjected to lateral and longitudinal slippage. Our controller has a better response time (Tr < 5 s) well as relatively low errors.

## CONCLUSION

In this paper, the design of a Fuzzy-FOPID controller is done for the problem of tracking the trajectory of a nonholonomic wheeled mobile robot subjected to longitudinal and lateral sliding. The proposed controller consists of a combination of a fractional order PID (FOPID) and a fuzzy logic controller (FLC). In this controller, the parameters $K_p$, $K_i$ and $K_d$ of the FOPID controller are determined by the FLC. The performances of the proposed controller were obtained by simulation on MATLAB/Simulink software. The results obtained in comparison with those obtained with Fuzzy-PID, PID and FOPID controllers show that the proposed controller has the best convergence rate, the best response and the best tracking accuracy. It will be important to see the behavior of the Fuzzy-FOPID controller facing the external perturbations on the mobile robot and also to integrate the neural network and to design an ANFIS-FOPID controller. These different points will constitute the main objective of our future work.

## CONTRIBUTION OF AUTHORS

Jean-Blaise Mvondo Zanga, Ph.D. student, is the main author of the paper. Contributed to the research of scientific references used to edit the introduction of the paper. He contributed to relevant improvements of the research work conducted and in the improvement of the simulation results. He carefully implemented revisions requested by his coauthors. Moreover, he contributed also to the preparation and edition of the manuscript of this paper according to JEEECCS Template.

Bertrand Moffo Lonla, senior lecturer, contributed to the choice of the modeling method used. He also helped on the choice of the Fuzzy-FOPIDF controller structure set up. He provided a great assistance for the research of solution to the problems raised by the principal supervisor of this research work.

Félix Paune, senior lecturer, helped to proofread the different parts of the article and to remove typing errors, to improve the translation of some sentences. He also made valuable suggestions for the choice of some parameters relevant parameters of the designed system. Moreover, he contributed also to the preparation and edition of the manuscript of this paper according to JEEECCS Template.

Léandre Nneme Nneme, Full Professor, he provided relevant guidelines for the research methodology, adopted in this paper. He contributed to the organization and supervision of the works presented in this paper, as well as to the copyright ethic verification of its whole content.

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