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Synthesis-based design of RC one-ports

Foster and Cauer Syntheses

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***Abstract* –This work, which may be used within the didactic activity, presents the synthesis-based design of RC one-ports. The determination of the impedance function, followed by the Foster I and Foster II, Cauer I and Cauer II syntheses, leads to the achievement of four equivalent circuits (RC one-ports). Simulating these circuits, with the help of Orcad programme, we obtain real frequency characteristics, identical throughout the frequency domain, in which the analysis is carried out.**

Key words: Foster synthesis, Cauer synthesis, one-ports.

I. INTRODUCTION

The work presents the synthesis-based design of the RC one-ports. Knowing the characteristic of the impedance function, the determination of the impedance function (real positive function) allows the performance of the four types of syntheses: Foster I and Foster II, Cauer I and Cauer II. The syntheses lead to the achievement of four equivalent RC one-ports. The syntheses carried out correctly followed by the simulation with the help of Orcad programme, lead to the achievement of four frequency characteristics, identical throughout the frequency domain, in which the analysis is being carried out.

The use of Mathcad programme allows the illustration, in logarithmical scale, of the real and ideal frequency characteristics of the impedance function determined.

II. DETERMINATION OF THE IMPEDANCE FUNCTION

The characteristic of the impedance function is given in figure 1.

As presented in figure 1, the values of the frequencies which are breaking points of the given characteristic are as follows:

$$0.4=\frac{2}{5};\quad 0.6=\frac{3}{5};\quad 0.8=\frac{4}{5};\quad 1$$

These value shall be pole-zeros of the searched function.

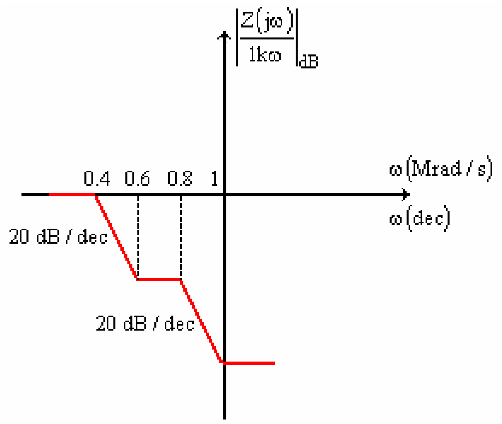

Figure 1. Caracteristic of the impedance function.

The impedance function shall be of the following type:

$$Z(s)=k\cdot\frac{\left(1+\frac{s}{s_1^{'}}\right)\left(1+\frac{s}{s_2^{'}}\right)}{\left(1+\frac{s}{s_1^{''}}\right)\left(1+\frac{s}{s_2^{''}}\right)} \qquad (1)$$

This function results based on the following reasoning: the prescribed line segments have an inclination of $20dB/dec$. This inclination is performed with factors of the following type: $\left(1+\frac{s}{s_i}\right)$.

The characteristic, logarithmically represented, is zero up to the frequency $s=0.4$. Starting with this frequency, the characteristic decreases by $20dB/dec$ to the frequency $s=0.6$. It is obtained with the help of a factor of type $\left(1+\frac{s}{s_1^{''}}\right)$, $\left(s_1^{''}=0.4\right)$, which has to be at the denominator of the function (pole), in order to ensure the decreasing character of the function.

Between the frequencies $s = 0.6$ and $s = 0.8$ the characteristic is a constant. It is obtained if a factor $\left(1+\frac{s}{s_1^{'}}\right), \left(s_1^{'} = 0.6\right)$ is introduced, which produces an ascending inclination by $20\text{dB}/\text{dec}$ and which is also aimed at annihilating the factor $\left(1+\frac{s}{s_1^{''}}\right)$. This factor has to be at the numerator of the function (zero), in order to ensure an ascending character.

Beginning with frequency $s = 0.8$, the prescribed frequency characteristic has a decreasing character again with an inclination of $20\text{dB}/\text{dec}$. This effect is obtained, as in the first case, introducing a factor of type $\left(1+\frac{s}{s_2^{''}}\right)$, $\left(s_2^{''} = 0.8\right)$ to the denominator of the function.

Beginning with frequency $s = 1$, the frequency characteristic has to remain constant. This effect is obtained introducing the factor $\left(1+\frac{s}{s_2^{''}}\right)$ to the numerator, which is aimed at annihilating the effect of the previously introduced factor.

Finally, we obtain function $Z(s)$ which has the following expression:

$$Z(s) = k \cdot \frac{\left(1+\frac{s}{3/5}\right)\left(1+\frac{s}{1}\right)}{\left(1+\frac{s}{4/5}\right)\left(1+\frac{s}{2/5}\right)} = k \cdot \frac{\left(1+\frac{5}{3}s\right)(1+s)}{\left(1+\frac{5}{4}s\right)\left(1+\frac{5}{2}s\right)} \quad (2)$$

The value of the constant k is determined by examining function $Z(s)$ at zero frequency.

From Bode diagram, presented in figure 1, it results that for $\omega = 0$, $\left|\frac{Z(0)}{1\text{k}\Omega}\right|_{\text{dB}} = 0\,\text{dB}$, $Z(0) = 1$.

From the relation of $Z(s)$ it results that: $Z(0) = k$, and therefore $k = 1$.

The final expression of function $Z(s)$ shall be:

$$Z(s) = \frac{8\left(s+\frac{3}{5}\right)(s+1)}{15\left(s+\frac{4}{5}\right)\left(s+\frac{2}{5}\right)} = \frac{40s^2+64s+24}{75s^2+90s+24} \quad (3)$$

III. FOSTER AND CAUER SYNTHESES

1. *Foster I Synthesis*

Foster I Synthesis of the RC one-port consists in developing $Z(s)$ in elementary fractions, relation (3), achieving each development term through an elementary one-port and connecting them in series:

$$Z(s) = k_\infty + \frac{k_1}{\left(s+\frac{4}{5}\right)} + \frac{k_2}{\left(s+\frac{2}{5}\right)} \quad (4)$$

Calculating, we obtain:

$$k_\infty = \lim_{s\to\infty} Z(s) = \frac{8}{15} \quad (5)$$

$$k_1 = \lim_{s\to-\frac{4}{5}} \left(s+\frac{4}{5}\right)\cdot Z(s) = \frac{4}{75} \quad (6)$$

$$k_2 = \lim_{s\to-\frac{2}{5}} \left(s+\frac{2}{5}\right)\cdot Z(s) = \frac{4}{25} \quad (7)$$

Finally, we obtain:

$$Z(s) = \frac{8}{15} + \frac{1}{\frac{75}{4}s+15} + \frac{1}{\frac{25}{4}s+\frac{5}{2}} \quad (8)$$

The diagram of Foster I-type RC one-port, according to the development (8), is presented in figure 2:

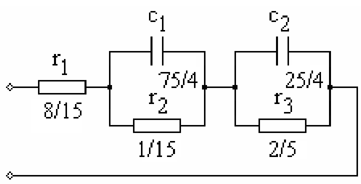

Figure 2. Foster I-type RC one-port

Observation: By performing the synthesis, the corresponding circuit will be formed of dimensionless norm circuit elements (r_k, c_k).

We obtain:

$$r_1 = \frac{8}{15};\ r_2 = \frac{1}{15};\ r_3 = \frac{2}{5}; \qquad c_1 = \frac{75}{4};\ c_2 = \frac{25}{4} \quad (9)$$

The return to the non-norm (R_k, C_k) is performed based on the relations:

$$r_k = \frac{R_k}{R_u}; \quad R_k = r_k \cdot R_u; \qquad c_k = \omega_u R_u C_k; \quad C_k = \frac{c_k}{\omega_u R_u} \quad (10)$$

For the synthesis of the one-ports, we consider as unit values of the elements:

$\omega_u = 1\,\text{Mrad/s};\quad R_u = 1\,\text{K}\Omega;\quad C_u = 1\,\text{nF}$

Calculating, we obtain the non-norm values:

$$R_1 = 0.533K\Omega; \quad C_1 = 18.75nF; \quad R_2 = 0.066K\Omega; \quad C_2 = 6.25nF; \quad R_3 = 0.4K\Omega \tag{11}$$

2. *Foster II Synthesis*

Foster II Synthesis of the RC one-port consists in developing $Y(s)/s$ in elementary fractions, clarifying $Y(s)$ in this development, achieving each fraction of $Y(s)$ through elementary one-ports and connecting them in parallel.

We obtain:

$$\frac{Y(s)}{s} = \frac{k_0}{s} + \frac{k_1}{s+\frac{3}{5}} + \frac{k_2}{s+1} \tag{12}$$

Calculating, we obtain:

$$k_0 = \lim_{s\to 0} s \cdot \frac{Y(s)}{s} = 1 \tag{13}$$

$$k_1 = \lim_{s\to -\frac{3}{5}} \left(s+\frac{3}{5}\right) \cdot \frac{Y(s)}{s} = \frac{5}{16} \tag{14}$$

$$k_2 = \lim_{s\to -1} (s+1) \cdot \frac{Y(s)}{s} = \frac{9}{16} \tag{15}$$

Finally, we obtain:

$$\frac{Y(s)}{s} = \frac{1}{s} + \frac{\frac{5}{16}}{\left(s+\frac{3}{5}\right)} + \frac{\frac{9}{16}}{(s+1)} \tag{16}$$

and: $Y(s) = 1 + \dfrac{1}{\frac{16}{5} + \frac{48}{25}\cdot\frac{1}{s}} + \dfrac{1}{\frac{16}{9} + \frac{16}{9}\cdot\frac{1}{s}}$ (17)

The diagram of Foster II-type RC one-port, according to the development (19), is presented in figure 3:

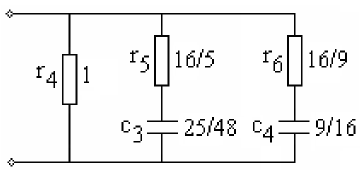

Figure 3. Foster II-type RC one-port

We obtain:

$$r_4 = 1; \; r_5 = \frac{16}{5}; \; r_6 = \frac{16}{9}; \quad c_3 = \frac{25}{48}; \; c_4 = \frac{9}{16} \tag{18}$$

The return to the non-norm values (R_k, C_k) is performed based on the relations (10).

Calculating, we obtain the non-norm values:

$$R_4 = 1K\Omega; \quad C_3 = 0.52nF; \quad R_5 = 3.2K\Omega; \quad C_4 = 0.562nF \quad R_6 = 1.777K\Omega \tag{19}$$

3. *Cauer I Synthesis*

When the degrees differ by one unit, the synthesis in Cauer I scale of the real positive RC functions does not pose any problems, the development beginning by dividing the higher degree polynomial by the lower degree polynomial and then continuing the development algorithm in continuous fraction.

For equal degrees, in the first stage we will extract a constant (a resistance), then in the 2nd stage we will extract the pole from the infinity.

Since only Y_{RC} (not Z_{RC}) can have pole at the infinity, it results that the function in the 2nd stage of the algorithm is an admittance, therefore in the first stage it has to be an impedance. Thus, Cauer I development applies to $Z(s)$ in this case, obtaining the continuous fraction:

$$Z(s) = \frac{40s^2+64s+24}{75s^2+90s+24} = \frac{40}{75} + \cfrac{1}{\frac{75}{16}s + \cfrac{1}{\frac{32}{75} + \cfrac{1}{\frac{625}{16}s + \frac{1}{25}}}} \tag{20}$$

The diagram of CAUER I-type RC one-port, according to the development (24), is presented in figure 4:

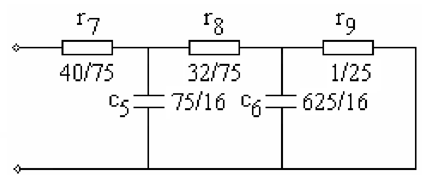

Figure 4. Cauer I-type RC one-port

We obtain:

$$r_7 = \frac{40}{75}; \; r_8 = \frac{32}{75}; \; r_9 = \frac{1}{25}; \quad c_5 = \frac{75}{16}; \; c_6 = \frac{625}{16} \tag{21}$$

The return to the non-norm values (R_k, C_k) is performed based on the relations (10).

Calculating, we obtain the non-norm values:

$$R_7 = 0.533K\Omega; \quad C_5 = 4.687nF; \quad R_8 = 0.426K\Omega; \quad C_6 = 39.062nF; \quad R_9 = 0.04K\Omega \tag{22}$$

4. *Cauer II Synthesis*

It does not pose any problems when the real positive RC function presents pole or zero in the origin, the algorithm beginning by dividing the polynomials with free term by the polynomial without a free term, obviously the polynomials are in ascending order.

When the function does not have pole in the origin, in the first stage we will extract a constant, then in the 2nd stage we will extract the pole from the origin. Since only Z_{RC} (not Y_{RC}) can have pole in the origin, it results that the function in the second stage of the algorithm is an impedance, therefore in the first stage it has to be an admittance. Therefore, Cauer II development applies to $Y(s)$ in this case, obtaining the continuous fraction:

$$Y(s)=\frac{75s^2+90s+24}{40s^2+64s+24} \tag{23}$$

$$Y(s)=\frac{24+90s+75s^2}{24+64s+40s^2}= \\ =1+\cfrac{1}{\cfrac{12}{13\cdot s}+\cfrac{1}{\cfrac{169}{206}+\cfrac{1}{\cfrac{42436}{2925\cdot s}+\cfrac{1}{\cfrac{45}{824}}}}} \tag{24}$$

The diagram of CAUER I-type RC one-port, according to the development (24), is presented in figure 5:

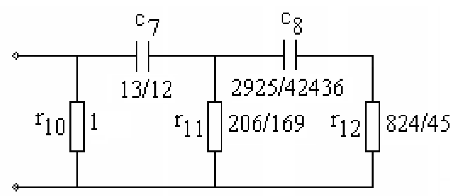

Figure 5. Cauer II-type RC one-port

We obtain:

$$r_{10}=1;\ r_{11}=\frac{206}{169};\ r_{12}=\frac{824}{45}; \\ c_7=\frac{13}{12};\ c_8=\frac{2925}{42436} \tag{25}$$

The return to the non-norm values (R_k, C_k) is performed based on the relations (10).

Calculating, we obtain the non-norm values:

$$R_{10}=1K\Omega;\quad C_7=1.083nF; \\ R_{11}=1.218K\Omega;\quad C_8=0.068nF; \\ R_{12}=18.311K\Omega \tag{26}$$

IV. Experiments

The illustration of the real and ideal frequency characteristics of the impedance function determined by relation (1), using Mathcad programme, is presented in figure 6.

$\omega := 0.01, 0.1 .. 100$

$\omega1 := 0.4 \quad \omega2 := 0.6 \quad \omega3 := 0.8 \quad \omega4 := 1$

$$a1(\omega) := -20\cdot\log\left(\frac{\omega}{\omega1}\right)\cdot\Phi(\omega-\omega1)$$

$$a2(\omega) := 20\cdot\log\left(\frac{\omega}{\omega2}\right)\cdot\Phi(\omega-\omega2)$$

$$a3(\omega) := -20\cdot\log\left(\frac{\omega}{\omega3}\right)\cdot\Phi(\omega-\omega3)$$

$$a4(\omega) := 20\cdot\log\left(\frac{\omega}{\omega4}\right)\cdot\Phi(\omega-\omega4)$$

$$ar1(\omega) := -10\cdot\log\left(1+\frac{\omega^2}{\omega1^2}\right)$$

$$ar2(\omega) := 10\cdot\log\left(1+\frac{\omega^2}{\omega2^2}\right)$$

$$ar3(\omega) := -10\cdot\log\left(1+\frac{\omega^2}{\omega3^2}\right)$$

$$ar4(\omega) := 10\cdot\log\left(1+\frac{\omega^2}{\omega4^2}\right)$$

$$a(\omega) := a1(\omega)+a2(\omega)+a3(\omega)+a4(\omega)$$

$$ar(\omega) := ar1(\omega)+ar2(\omega)+ar3(\omega)+ar4(\omega)$$

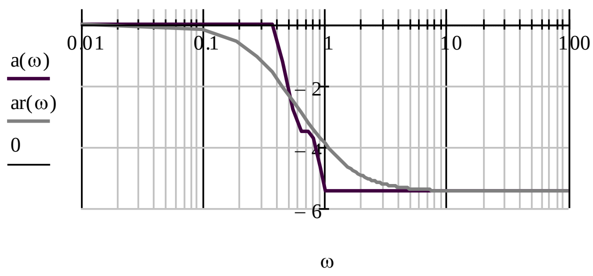

Figure 6. The real and ideal frequency characteristics of the impedance function

To simulate Foster and Cauer-type RC one-ports we use Orcad programme.

In figure 7 we present the circuits of Foster and Cauer-type RC one-ports, simulated in Orcad

The real frequency characteristics obtained by simulating the circuits of Foster and Cauer-type RC one-ports in Orcad are presented in figure 8.

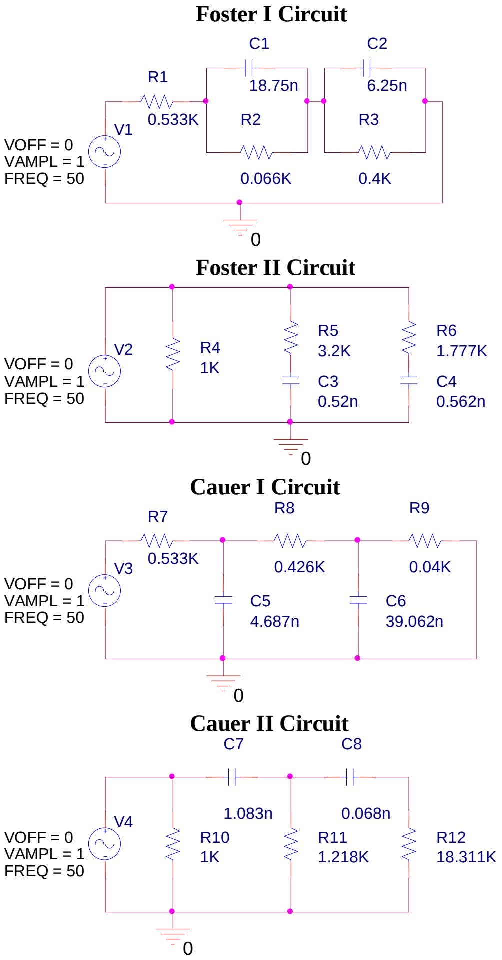

Figure 7. Circuits of Foster and Cauer-type one-ports

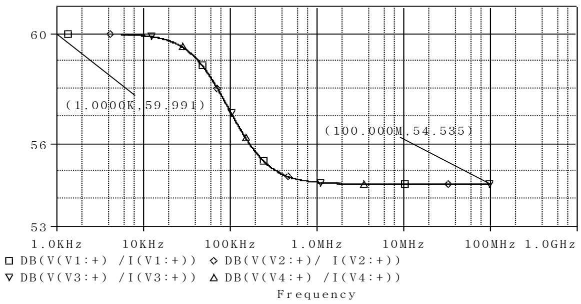

Figure 8. Real frequency characteristics

V. Conclusions

The correct determination of the impedance function, followed by the correct performance of the Foster and Cauer syntheses lead to the achievement of equivalent RC one-port circuits.

The correct simulation of these circuits in Orcad allows the achievement of real frequency characteristics, identical throughout the frequency domain, in which the analysis is being carried out.

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